Numeric Response Questions

Permutations and Combinations

Q.1 Find the number of ways in which 20 one rupee coins can be distributed among 5 people such that each person gets atleast 3 rupees.

Q.2 In a group of 13 cricket players, four are bowlers. Find out in how many ways can they form a cricket team of 11 players in which at least 2 bowlers are included.

Q.3 A teacher takes three children from her class to the zoo at a time, but she does not take the same children to the zoo more than once. She finds that she went to the zoo 84 times more than a particular child has gone to the zoo. Find the number of children in her class.

Q.4 Find the number of words of four letters that can be formed from the letters of the word 'EXAMINATION'.

Q.5 There are 6 letters and 6 directed envelopes, Find the number of ways in which all letters are put in the wrong envelopes.

Q.6 How many different nine-digit numbers can be formed from the digits of the number 223355888 by rearrangement of the digits so that odd digits occupy even places?

Q.7 Find the total number of 4 digit number that are greater than 3000, that can be formed using the digits 1,2,3,4,5,6 (no digit is being repeated in any number).

Q.8 Find the number of arrangements of letters of the word BANANA in which the two N's do not appear adjointly,

Q.9 Find the number of five digit telephone numbers having atleast one of their digits repeated.

Q.10 In a chess tournament, all participant were to play one game with the other. Two players fell ill after having played 6 games (they had not played with each other). If total number of games played in the tournament is equal to 84, then find the total number of participants in the beginning.

Q.11 A committee of 5 is to be formed from 9 ladies and 8 men. If the committee commands a lady majority, then find the number of ways this can be done.

Q.12 The letters of the word MODESTY are written in all possible orders and these words are written out as in a dictionary, then find rank of word MODESTY.

Q.13 Find total number of divisors of 480 , that are of the form $4n + 2, n \geq 0$.

Q.14 Find the number of zeroes at the end of 300 !.

Q.15 Find the number of triangles whose vertices are at the vertices of an octagon but none of whose sides happen to come from the sides of the octagon.

ANSWER KEY

1. 105.00	**2.** 78.00	**3.** 10.00	**4.** 2454.00	**5.** 265.00	**6.** 60.00	**7.** 240.00
8. 40.00	**9.** 69760.00	**10.** 15.00	**11.** 3486.00	**12.** 1681.00	**13.** 4.00	**14.** 74.00
15. 16.00						

Hints & Solutions

1. 5 people each get atleast 3 rupees so left Rs. 5 is to be distributed

$x_1 + x_2 + x_3 + x_4 + x_5 = 5$

No. of ways $= {}^{5+5-1}C_{5-1}$

${}^{9}C_4 = 105$

2.

Bowler (4)	Rest (9)
2	9
3	8
4	7

$\Rightarrow {}^{4}C_2 \times {}^{9}C_9 + {}^{4}C_3 \times {}^{9}C_8 + {}^{4}C_4 \times {}^{9}C_7$

$= 6 + 36 + 36 = 78$

3. ${}^{n}C_3 - {}^{n-1}C_2 = 84$

$\frac{(n-1)(n-2)}{6}[n-3] = 84$

$\Rightarrow (n-1)(n-2)(n-3) = 9 \times 8 \times 7$

$\Rightarrow n = 10$

4. (A A), (I I), (N N), E, X, M, T, O

Case–1 : All 4 different.

${}^{8}C_4 \times 4! = 1680$

Case–2 : 2 Alike, 2 different

$3 \times {}^{7}C_2 \times \frac{4!}{2!} = 756$

Case– 3 : 2 alike of 1 kind, 2 alike of 2nd kind.

$({}^{3}C_2 \times 1 \times 1) \times \frac{4!}{2!\,2!} = 18$

Total = 1680 + 756 + 18 = 2454

5. $D_6 = 6!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right]$

$= 360 - 120 + 30 - 6 + 1 = 391 - 126 = 265$

6. 22888

3355

$\frac{4!}{2!\,2!} \times \frac{5!}{2!\,3!} \Rightarrow \frac{24}{2.2} \times \frac{120}{2.6}$

$\Rightarrow 6 \times 10 = 60$

7. First place can be filled by 3, 4, 5, 6

So total ways $= 4 \times 5 \times 4 \times 3 = 240$

8. No. of words in which two N are not together

$\frac{6!}{3!2!} - \frac{5!}{3!} = 40$

9. No. of numbers with atleast one digit repeated

$= 10^5 - 10 \times 9 \times 8 \times 7 \times 6 = 69760$

10. n $\xrightarrow{2x}$ (n–2)

$\Rightarrow {}^{n-2}C_2 + 6 = 84 \Rightarrow {}^{n-2}C_2 = 78$

$\Rightarrow (n-2)(n-3) = 156 = 12 \times 13$

$n - 2 = 13 \rightarrow n = 15$

11.

9L	8M	
5	0	
4	1	
3	2	$\rightarrow 5$

$= {}^{9}C_5 + {}^{9}C_4.{}^{8}C_1 + {}^{9}C_3.{}^{8}C_2$

$= \frac{9.8.7.6}{24} + \frac{9.8.7.6}{24} \times 8 + \frac{9.8.7}{6} \times 28$

$= 9.14 + 14 \times 9 \times 8 + 84 \times 28$

$= 3486$

12.

M	O	D	E	S	T	Y
$\begin{pmatrix}6!\\ \times\\ 2\end{pmatrix}$	$\begin{pmatrix}5!\\ \times\\ 2\end{pmatrix}$	$\begin{pmatrix}4!\\ \times\\ 0\end{pmatrix}$	$\begin{pmatrix}3!\\ \times\\ 0\end{pmatrix}$	$\begin{pmatrix}2!\\ \times\\ 0\end{pmatrix}$	$\begin{pmatrix}1!\\ \times\\ 0\end{pmatrix}$	$\begin{pmatrix}0!\\ \times\\ 1\end{pmatrix}$

$1440 + 240 + 0 + 0 + 0 + 0 + 1 = 1681$

13. $480 = 2^5 \times 3^1 \times 5^1$

(4n + 2) divisor = Groups taking exactly one 2

$= 1 \times 2 \times 2$

$= 4$

14. $E_2 = (300!)$

$$= \left[\frac{300}{2}\right] + \left[\frac{300}{2^2}\right] + \left[\frac{300}{2^3}\right] + \left[\frac{300}{2^4}\right] + \left[\frac{300}{2^5}\right] + \left[\frac{300}{2^6}\right] + \left[\frac{300}{2^7}\right] + \left[\frac{300}{2^8}\right] + \ldots$$

$E_2(300!) = 150 + 75 + 37 + 18 + 9 + 4 + 2 + 1$

$= 296$

$$E_5(300!) = \left[\frac{300}{5}\right] + \left[\frac{300}{5^2}\right] + \left[\frac{300}{5^3}\right] + \ldots\ldots\ldots$$

$= 60 + 12 + 2 = 74$

No. of zeroes = 74

15. ${}^8C_3 - 8(8 - 4) - 8$

$= 56 - 32 - 8 = 16$